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Restoration of Electro–Optically Induced Phase Shift of Light Passed Through the Liquid Crystal

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New method of restoring the phase shift of the light passing through the nematic liquid crystal (NLC) has been developed. Hilbert transformation supported by Unwrap *function of Matlab software is proposed to be used for recovering phase shift from experimentally obtained data of light intensity vs. applied voltage. The method makes the phase shift dependence less sensitive to measurement noises.*

Keywords Clearing temperature; electro-optically induced birefringence; Hilbert transformation; nematic liquid crystal; phase shift

Introduction

To use electrically controlled LC phase retarders in different applications, it is necessary to have the phase shift calibration curve as a function of external voltage. The method based on the registration of light intensity vs. control voltage is considered to be the most convenient one.

Any variation of electric field applied to LC phase retarder leads to changes of electro-optically induced birefringence, and thus changes the light intensity and phase, in response. In experiments, the dependence of light intensity on applied voltage is recorded. A critical feature of phase retarders – phase shift vs. applied voltage function is, usually, restored from experimentally measured light intensity oscillations observed on the intensity vs. voltage curve. Obviously, the accuracy of determination of oscillation frequency is directly dependent on extent of precision of determination of oscillation coordinate. Proposed in this work, a novel method of restoration of phase shift vs. voltage dependence employs the Hilbert transformation and it is shown that this method makes the sought dependence less sensitive to measurement noises.

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Electro-Optically Induced Phase Shift of Light, Passed Through NLC Retarder

At normal incidence of the linearly polarized laser beam on the planar-oriented NLC cell with the director orientation at angle 45° in respect to the beam polarization plane in anisotropic LC medium, two orthogonal-polarized waves of different phase velocities appear [1].

Phase shift between these waves passing through LC layer with thickness d, is determined by

$$\Delta\Gamma(T,V) = \frac{2\pi}{\lambda_0} d\Delta n(T,V), \tag{1}$$

where λ_0 is the wavelength of the light in the vacuum, n_0 and n_e are LC refractive indexes for ordinary and extraordinary waves correspondingly.

In general, the following expression is used to determine the intensity of the light passing through the polarizer – LC cell – analyzer system with angle of 90° between axes of polarizer and analyzer [2]:

$$I(T,V) = I_0 \sin^2\left(\frac{\Delta\Gamma(T,V)}{2}\right), \tag{2}$$

where I_0 is laser beam intensity at the input of the system.

Having recorded the ratio I/I_0, one can calculate the appropriate phase shift using the following formula:

$$\Delta\Gamma = \pm 2\arcsin\sqrt{I/I_0} \tag{3}$$

It is worthy to note, that an algorithm of phase unwrapping is used to achieve the continuity of restorable phase with each phase shift, exceeding 2π.

When registering I/I_0 vs. applied voltage, the measurement noise occurs. To decrease the level of measurement noise it is necessary to conduct some preliminary processing of experimental data. It should be taken into account that values of applied external voltage corresponding to the same value of light intensity, will range between $10\Delta\ V$ to $20\ \Delta V$, where ΔV is a single step of voltage variation. Hence, for median filtration of data, the number of samples, used in averaging should be within 10 to 20, and filtration should be carried out in accordance with

$$\bar{y}(n) = \sum_{k=-N/2}^{N/2-1} \frac{y(n-k)}{N}, \tag{4}$$

where N is in the range from 10 to 20.

After the filtration had done, we start to restoration the phase dependence. From the function

$$\cos(\Delta\Gamma) = 1 - 2\sin^2\left(\frac{\Delta\Gamma}{2}\right) = 1 - 2I/I_0, \tag{5}$$

one can get

$$\cos(\Delta\Gamma) + j\sin(\Delta\Gamma), \tag{6}$$

applying the Hilbert transformation. In other words, to describe the phase shift dependence vs. applied voltage, instead of real signal $\cos(\Delta\Gamma)$, we use the analytical signal $\exp(j\Delta\Gamma)$ having obtained due to Hilbert transformation. Then, the phase shift dependence can be recovered by

$$\Delta\Gamma = \arctan(\sin(\Delta\Gamma)/\cos(\Delta\Gamma)) \tag{7}$$

Because of the periodicity of the arctan function, the phase is not continuous and has jumps of 2π. This can be corrected by phase unwrapping with the following simple algorithm:

for $i = 2{:}\, length(\Delta\Gamma)$
$\Delta\Gamma(i) = \Delta\Gamma(i) - ruond((\Delta\Gamma(i) - \Delta\Gamma(i-1))/(2\pi))2\pi;$
end,

where $\Delta\Gamma$ is an array containing the phase data obtained from the experimentally registered dependence of light intensity vs. applied voltage, $length(\Delta\Gamma)$ is the length of array $\Delta\Gamma$, *round* is round-off operator. Applying the Matlab function $unwrap(a\tan 2(\sin(\Delta\Gamma), \cos(\Delta\Gamma)))$, we find the phase shift $\Delta\Gamma$. The function *unwrap* is applied for providing the continuity of the recovering phase shifts exceeding 2π [3].

It should be noted that for investigation of induced birefringence the phase value at voltages less than the threshold is not of practical interest. Therefore, during the recovering, the values of phase, corresponding to voltages bellow threshold are reduced to zero, and the calculation of phase shift is started from the zero.

To verify the proposed algorithm the temperature dependence of electro-optically induced birefringence for 6CHBT (Poland) NLC was determined from the experimentally restored phase shift vs. temperature dependence. Below we present experimental results of the restoration of phase shift of the light, passed through the polarizer – NLC – analyzer system, for various cell temperatures.

The Experimental Setup and the Measurement Procedure

Cells with planar aligned 7 μm thick liquid crystal 6CHBT, were prepared. 1 mm thick glass plates, coated by 20 nm thick transparent conducting ITO layer, were used as substrates. The orientation of LC molecules on the substrates was performed by exposure of alignment layer ROP-103 LPP (ROLIC) by linearly polarized radiation of *He-Cd* 325 nm laser (KIMMON). The schematic of the experimental setup is shown in Figure 1.

The radiation of *He-Ne* cw laser (1) at wavelength 633 nm is directed to the NLC cell (3) through the polarizer (2). Director orientation is set 45° relative to polarization plane of the input polarizer (2). The Glan prism (4) serves as an analyzer. The investigated NLC cell was placed in the temperature setter, by means of which the cell temperature is kept with the accuracy ±0.25° C. The measurements were made at 11°C, 20°C, 30°C and 44°C. The laser beam were detected by a photodetector (5) connected to PC via NI DAQ 6025E. The parameters of applied voltage were controlled by ADC and specific LabView based program, allowing to specify the voltage pulse shape, as well as to change their amplitude in accordance with the given algorithm, with 50 mV steps in the range of ±10 V. The pulse frequency was 2000 Hz and duty cycle–0.5. The pulses after amplification were applied to LC cell. The control voltage variation rate was chosen to allow the cell to reach its steady state.

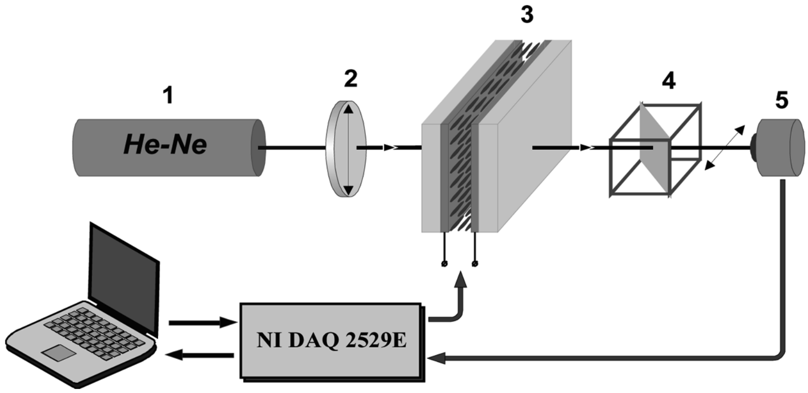

Figure 1. Experimental setup.

The experimental dependence of planar-oriented LC cell transmission vs. the amplitude of external rectangular bipolar voltage pulses (amplitude variation rate is 17 mV/s) at 20°C, is presented in Figure 2.

On the basis of obtained dependence, using the Hilbert transformation and algorithm of phase unwrapping, the phase shift curves at 11°C, 20°C, 30°C, and 44°C were restored (Fig. 3).

According to the given dependences the phase shift, caused by electro-optically induced birefringence, decreases with increasing the temperature. The phase dependences restoration was carried out using the algorithm, described in the previous chapter.

To carry out a verification of proposed method we determined clearing point of liquid crystal 6CHBT by calculating temperature dependence of birefringence for various applied voltages from data of Figure 3. In Figure 4 the values of experimentally obtained birefringence are marked by dots and the result of interpolation is shown by a continuous curve. The maximum value of the approximating relative error does not exceed 3%.

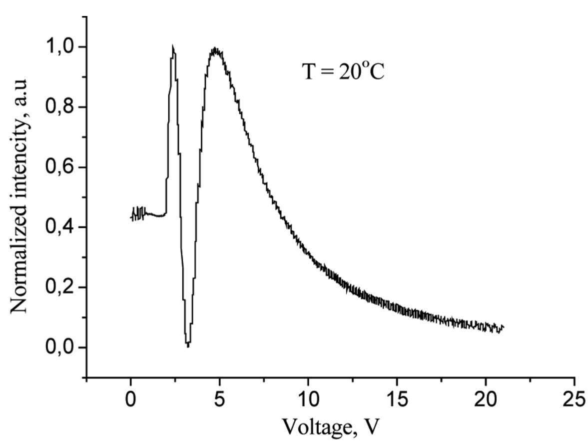

Figure 2. Light intensity as a function of applied voltage.

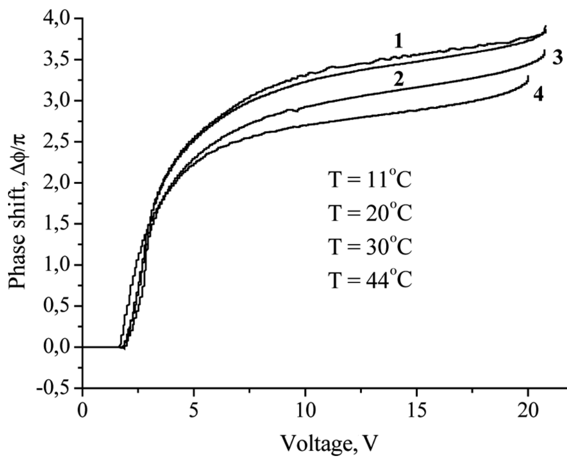

Figure 3. Phase shift vs. applied voltage at different temperatures.

To approximate of induced birefringence vs. temperature function, using obtained data the following formula was used:

$$\Delta n(T) = n_0 + n_1 \exp\left\{-\frac{T}{\tau_1}\right\} + n_2 \exp\left\{-\frac{T}{\tau_2}\right\} \tag{8}$$

The values of n_0, n_1, and n_2 for different bias voltages are given in Table 1.

On the base of exponential approximation (8) the clearing temperature was obtained from the equation $d\Delta n(T)/dT = 0$ for 6CHBT NLC is 70°C. But according to our experimental results all curves in Figure 4 coincided about at 320 K which corresponds to the clearing temperature value 47°C. The obtained value of clearing point is in good agreement with the value presented in [4].

Table 1. Approximation coefficients of refraction index, obtained on the base of experiment

V, (V)	n_0	n_1	n_2	τ_1	τ_2
7.5	0.04607	−6.99712e-10	−6.99712e-10	−19.77966	−19.77966
8.5	0.04964	−2.03513e-8	−2.03513e-8	−24.6981	−24.6981
9.5	0.05117	−1.9955e-8	−1.9955e-8	−24.56751	−24.56751
10.5	0.05258	−2.21731e-8	−2.21731e-8	−24.67866	−24.67866
11.5	0.05439	−6.85348e-8	−6.85348e-8	−26.88776	−26.88776
12.5	0.05355	−8.2629e-9	−8.2629e-9	−22.93008	−22.93008
13.5	0.055	−2.30717e-8	−2.30717e-8	−24.66198	−24.66198
14.5	0.05571	−2.70046e-8	−2.70046e-8	−24.9482	−24.9482
15.5	0.05566	−1.02438e-8	−1.02438e-8	−23.23441	−23.23441
16.5	0.05623	−1.00485e-8	−1.00485e-8	−23.20271	−23.20271
17.5	0.05613	−3.2779e-9	−3.2779e-9	−21.51502	−21.51502

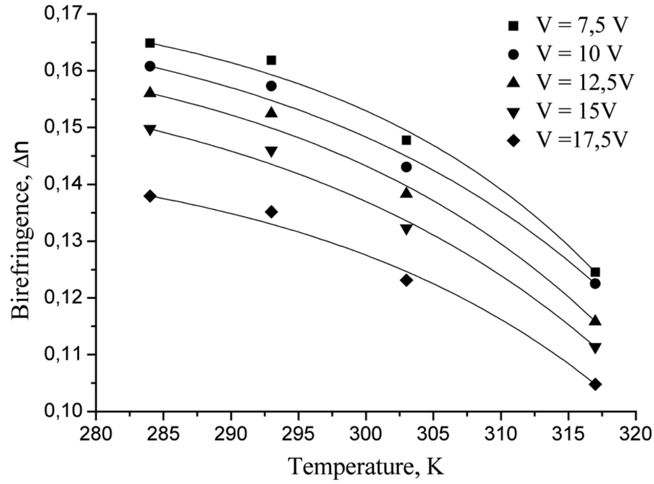

Figure 4. Induced birefringence as a function of temperature at different voltages.

Conclusion

A novel method of restoration of phase shift vs. voltage dependence using the Hilbert transformation is considered. It is shown that this method can be successfully used to eliminate measurement noises while calculating phase shift.

To test the proposed algorithm the temperature dependence of electro-optically induced birefringence for 6CHBT NLC was determined from the experimentally restored phase shift vs. temperature dependence. For NLC 6CHBT the clearing temperature was found to be ~47°C.

Acknowledgments

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References

[1] Blinov, L. M., & Chigrinov, V. G. (1994). *Electrooptic Effects in Liquid Crystal Materials*, Springer-Verlag: New York.
[2] Khoo, C., & Wu, S. T. (1993). *Optics and Nonlinear Optics of Liquid Crystals, World Scientific*:Singapore.
[3] Stearns D., Samuel & Ruth A., David. (1996). *Signal Processing Algorithms in Matlab*, Prentice Hall P T R, Upper Saddle River: New Jersey.
[4] Serak, S., Tabiryan, N., & Zeldovich, B. (2007). *Opt. Letters*, *32*(2), 169–171.